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Economic and Deterministic Quantum Teleportation of Arbitrary Bipartite Pure and Mixed State with Shared Cluster Entanglement

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Abstract We present a novel protocol for teleportation of arbitrary bipartite pure and mixed state with shared cluster entanglement in this paper. By employing Bell-state measurement on the teleported state and the shared cluster state twice, a sender could transmit the arbitrary bipartite state to a distant receiver. We show the good feature of the cluster state channel, with which it can realize the deterministic teleportation rather than probabilistic one. Moreover, since we require less particles to be shared and need no auxiliary qubit in our protocol, it is more efficient and applicable than the previous schemes.

Keywords Quantum teleportation · Bipartite state · Cluster entanglement

1 Introduction

Quantum mechanics allows many profound applications in the field of information science [1]. Since the discovery of quantum teleportation made by Bennett et al. in 1993 [2], it has attracted a lot of attention in the research direction. In 1998, Karlsson and Bourennane [3] proposed a quantum teleportation protocol with three-particle entangled states. Luo and Guo introduced a new way for teleporting bipartite entangled state $x_1|00\rangle + x_2|11\rangle$ in 2000 [4]. Later on, resort to the four-particle Greenberger-Horne-Zeilinger (GHZ) state $\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$, Lee present another way for teleportation of bipartite entangled state $x_1|01\rangle + x_2|10\rangle$ [5]. And recently, Deng et al. have proposed a symmetric multiparty-controlled teleportation protocol for an arbitrary two-particle entangled pure state in which

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two three-particle GHZ states are used [6]. Though it can realize the arbitrary bipartite entangled pure state teleportation in the end, the efficiency is comparatively low since each quantum teleportation involves six shared particles.

Motivated by the work of [7], we want to introduce a novel protocol to teleport arbitrary bipartite entangled state, no matter whether it is pure or mixed. The previous protocol [7] used the cluster state to realize controlled teleportation of single particle state. The cluster state has such a good feature that it is harder to be destroyed by local operation than GHZ-class state and it has properties of both GHZ-class and W-class states [8, 9]. With Bell-state measurement on the teleported state and the shared cluster state twice, our protocol show its realization of arbitrary bipartite teleportation with this kind of entanglement. In the following, we detail our protocol and then calculate the entanglement of shared cluster state to estimate its capability for quantum teleportation.

2 Quantum Teleportation of Arbitrary Bipartite State

Now we discuss our protocol in details. Firstly, we present the method to teleport arbitrary bipartite pure state of system xy. The state to be teleported can be represented as:

$$|\Phi\rangle_{xy} = x_1|00\rangle + x_2|01\rangle + x_3|10\rangle + x_4|11\rangle, \tag{1}$$

where $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$.

Suppose the sender Alice and the receiver Bob share one of the following four-particle cluster states [10] as the quantum channel:

$$\begin{aligned}
|Cluster^{\pm}\rangle^{\boldsymbol{1}}_{1234} &= \frac{1}{2}(\pm|0000\rangle + |0011\rangle + |1100\rangle \mp |1111\rangle)_{1234},\\
|Cluster^{\pm}\rangle^{\boldsymbol{2}}_{1234} &= \frac{1}{2}(|0000\rangle \pm |0011\rangle + |1100\rangle \mp |1111\rangle)_{1234},\\
|Cluster^{\pm}\rangle^{\boldsymbol{3}}_{1234} &= \frac{1}{2}(\pm|0001\rangle \mp |0010\rangle + |1101\rangle + |1101\rangle)_{1234},\\
|Cluster^{\pm}\rangle^{\boldsymbol{4}}_{1234} &= \frac{1}{2}(|0001\rangle + |0010\rangle \pm |1101\rangle \mp |1101\rangle)_{1234},\\
|Cluster^{\pm}\rangle^{\boldsymbol{5}}_{1234} &= \frac{1}{2}(\pm|0100\rangle + |0111\rangle \mp |1000\rangle + |1011\rangle)_{1234},\\
|Cluster^{\pm}\rangle^{\boldsymbol{6}}_{1234} &= \frac{1}{2}(|0100\rangle \pm |0111\rangle + |1000\rangle \mp |1011\rangle)_{1234},\\
|Cluster^{\pm}\rangle^{\boldsymbol{7}}_{1234} &= \frac{1}{2}(\pm|0101\rangle + |0110\rangle + |1001\rangle \mp |1010\rangle)_{1234},\\
|Cluster^{\pm}\rangle^{\boldsymbol{8}}_{1234} &= \frac{1}{2}(|0101\rangle \pm |0110\rangle \mp |1001\rangle + |1010\rangle)_{1234},
\end{aligned} \tag{2}$$

where the particle $1, 3$ are kept by Alice and particle $2, 4$ by Bob. For definiteness, we assume Alice and Bob share the $|Cluster^{+}\rangle^{\boldsymbol{1}}$ state:

$$|Cluster^{+}\rangle^{\boldsymbol{1}}_{1234} = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle). \tag{3}$$

Then the state of the composite quantum system composed of the six particles x, y, 1, 2, 3 and 4 can be written as

$$\begin{aligned}|\Psi\rangle_{xy1234} &= |\Phi\rangle_{xy} \otimes |Cluster\rangle_{1234} \\ &= \frac{1}{2}[|\phi^+\rangle_{x1}(x_1|0000\rangle + x_2|1000\rangle + x_1|0011\rangle + x_2|1011\rangle \\ &\quad + x_3|0100\rangle + x_4|1100\rangle - x_3|0111\rangle - x_4|1111\rangle)_{y234} \\ &\quad + |\phi^-\rangle_{x1}(x_1|0000\rangle + x_2|1000\rangle + x_1|0011\rangle + x_2|1011\rangle \\ &\quad - x_3|0100\rangle - x_4|1100\rangle + x_3|0111\rangle + x_4|1111\rangle)_{y234} \\ &\quad + |\psi^+\rangle_{x1}(x_1|0100\rangle + x_2|1100\rangle - x_1|0111\rangle - x_2|1111\rangle \\ &\quad + x_3|0000\rangle + x_4|1000\rangle + x_3|0011\rangle + x_4|1011\rangle)_{y234} \\ &\quad + |\psi^-\rangle_{x1}(x_1|0100\rangle + x_2|1100\rangle - x_1|0111\rangle - x_2|1111\rangle \\ &\quad - x_3|0000\rangle - x_4|1000\rangle - x_3|0011\rangle - x_4|1011\rangle)_{y234}],\end{aligned} \tag{4}$$

where $|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$, $|\psi^-\rangle$ are the four two particle Bell-states, which are

$$\begin{aligned}|\phi^\pm\rangle &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \\ |\psi^\pm\rangle &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle).\end{aligned} \tag{5}$$

When Alice performs Bell-state measurement on particles x and 1, the state composed of particles y, 2, 3 and 4 would collapse onto one of the following four states corresponding to the four Bell-states with equal probability $\frac{1}{4}$:

$$\begin{aligned}|\Psi\rangle^{\boldsymbol{1}}_{y234} &= \frac{1}{\sqrt{2}}(x_1|0000\rangle + x_2|1000\rangle + x_1|0011\rangle + x_2|1011\rangle \\ &\quad + x_3|0100\rangle + x_4|1100\rangle - x_3|0111\rangle - x_4|1111\rangle)_{y234},\end{aligned} \tag{6}$$

$$\begin{aligned}|\Psi\rangle^{\boldsymbol{2}}_{y234} &= \frac{1}{\sqrt{2}}(x_1|0000\rangle + x_2|1000\rangle + x_1|0011\rangle + x_2|1011\rangle \\ &\quad - x_3|0100\rangle - x_4|1100\rangle + x_3|0111\rangle + x_4|1111\rangle)_{y234},\end{aligned} \tag{7}$$

$$\begin{aligned}|\Psi\rangle^{\boldsymbol{3}}_{y234} &= \frac{1}{\sqrt{2}}(x_1|0100\rangle + x_2|1100\rangle - x_1|0111\rangle - x_2|1111\rangle \\ &\quad + x_3|0000\rangle + x_4|1000\rangle + x_3|0011\rangle + x_4|1011\rangle)_{y234},\end{aligned} \tag{8}$$

$$\begin{aligned}|\Psi\rangle^{\boldsymbol{4}}_{y234} &= \frac{1}{\sqrt{2}}(x_1|0100\rangle + x_2|1100\rangle - x_1|0111\rangle - x_2|1111\rangle \\ &\quad - x_3|0000\rangle - x_4|1000\rangle - x_3|0011\rangle - x_4|1011\rangle)_{y234}.\end{aligned} \tag{9}$$

After the measurement, Alice publishes her measurement outcomes. And when the outcomes are $|\phi^-\rangle$, $|\psi^+\rangle$ or $|\psi^-\rangle$, with the cooperation of Alice's and Bob's unitary transformations $\sigma_z^2 \otimes I^3$, $\sigma_x^2 \otimes \sigma_z^3$ or $i\sigma_y^2 \otimes \sigma_z^3$ on particle 2 and 3 respectively, the state of particle

y, 2, 3 and 4 can be transformed to $|\Psi\rangle_{y234}^{\boldsymbol{1}}$ precisely:

$$|\Psi\rangle_{y234}^{\boldsymbol{1}} = \sigma_z^2 \otimes I^3 |\Psi\rangle_{y234}^{\boldsymbol{2}} = \sigma_x^2 \otimes \sigma_z^3 |\Psi\rangle_{y234}^{\boldsymbol{3}} = i\sigma_y^2 \otimes \sigma_z^3 |\Psi\rangle_{y234}^{\boldsymbol{4}}. \tag{10}$$

To present the next procedure more clearly, we rewrite the state $|\Psi\rangle_{y234}^{\boldsymbol{1}}$ as follows:

$$\begin{aligned}
|\Psi\rangle_{y324}^{\boldsymbol{1}} &= \frac{1}{\sqrt{2}}(x_1|0000\rangle + x_2|1000\rangle + x_1|0101\rangle + x_2|1101\rangle \\
&\quad + x_3|0010\rangle + x_4|1010\rangle - x_3|0111\rangle - x_4|1111\rangle)_{y324} \\
&= \frac{1}{\sqrt{2}}[0_y 0_3(x_1|00\rangle + x_3|10\rangle)_{24} + 0_y 1_3(x_1|01\rangle - x_3|11\rangle)_{24} \\
&\quad + 1_y 0_3(x_2|00\rangle + x_4|10\rangle)_{24} + 1_y 1_3(x_2|01\rangle - x_4|11\rangle)_{24}] \\
&= \frac{1}{2}|\phi^+\rangle_{y3}(x_1|00\rangle + x_2|01\rangle + x_3|10\rangle - x_4|11\rangle)_{24} \\
&\quad + \frac{1}{2}|\phi^-\rangle_{y3}(x_1|00\rangle - x_2|01\rangle + x_3|10\rangle + x_4|11\rangle)_{24} \\
&\quad + \frac{1}{2}|\psi^-\rangle_{y3}(x_1|01\rangle + x_2|00\rangle - x_3|11\rangle + x_4|10\rangle)_{24} \\
&\quad + \frac{1}{2}|\psi^-\rangle_{y3}(x_1|01\rangle - x_2|00\rangle - x_3|11\rangle - x_4|10\rangle)_{24}.
\end{aligned} \tag{11}$$

When they have cooperated to transform the state of particle y, 2, 3 and 4 into the state $|\Psi\rangle_{y234}^{\boldsymbol{1}}$ (or equally $|\Psi\rangle_{y324}^{\boldsymbol{1}}$), Alice performs the second Bell-state measurement on particles y and 3, resulting in the state of particles 2 and 4 collapsing onto one of four states in the following also with equal probability $\frac{1}{4}$:

$$|\Upsilon\rangle_{24}^{\boldsymbol{A}} = (x_1|00\rangle + x_2|01\rangle + x_3|10\rangle - x_4|11\rangle)_{24}, \tag{12}$$

$$|\Upsilon\rangle_{24}^{\boldsymbol{B}} = (x_1|00\rangle - x_2|01\rangle + x_3|10\rangle + x_4|11\rangle)_{24}, \tag{13}$$

$$|\Upsilon\rangle_{24}^{\boldsymbol{C}} = (x_1|01\rangle + x_2|00\rangle - x_3|11\rangle + x_4|10\rangle)_{24}, \tag{14}$$

$$|\Upsilon\rangle_{24}^{\boldsymbol{D}} = (x_1|01\rangle - x_2|00\rangle - x_3|11\rangle - x_4|10\rangle)_{24}. \tag{15}$$

After that, Alice publishes her measurement outcomes again. Since the particle 2 and 4 are both in Bob's hand, if he wants to reconstruct the initial teleported state, all he has to do is to employ the transformation gate U^A, U^B, U^C or U^D on the two particles according to the result of Alice's measurement. Here,

$$U^{\mathbf{A}}|\Upsilon\rangle^{\mathbf{A}} = U^{\mathbf{B}}|\Upsilon\rangle^{\mathbf{B}} = U^{\mathbf{C}}|\Upsilon\rangle^{\mathbf{C}} = U^{\mathbf{D}}|\Upsilon\rangle^{\mathbf{D}} = |\Phi\rangle, \tag{16}$$

and the four transformation gates are

$$U^{\boldsymbol{A}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{17}$$

$$U^{\boldsymbol{B}}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix},\tag{18}$$

$$U^{\boldsymbol{C}}=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix},\tag{19}$$

$$U^{\boldsymbol{D}}=\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix}.\tag{20}$$

Till now, the teleportation of arbitrary bipartite pure state from Alice to Bob has been accomplished. We would like to generalize it to the case of arbitrary bipartite mixed state. Consider a bipartite mixed state ρ of system xy, which can be decomposed as [11, 12]:

$$\rho_{xy}=\sum_{i=1}^{4} P_i\ |\Phi_i\rangle_{xy}\ \langle\Phi_i|,\tag{21}$$

where $0\le P_i\le 1$, $\sum_{i=1}^{4} P_i=1$ and $|\Phi_i\rangle$ are bipartite pure states with the same entanglement of formation as ρ. In order to display the protocol well, we set the state as:

$$|\Phi_{mixed}\rangle_{xy}=\begin{pmatrix}|\Phi_1\rangle_{xy} & |\Phi_2\rangle_{xy} & |\Phi_3\rangle_{xy} & |\Phi_4\rangle_{xy}\\ P_1 & P_2 & P_3 & P_4\end{pmatrix},\tag{22}$$

where $|\Phi_i\rangle_{xy}=x_{1i}|00\rangle+x_{2i}|01\rangle+x_{3i}|10\rangle+x_{4i}|11\rangle$ and $x_{1i}^2+x_{2i}^2+x_{3i}^2+x_{4i}^2=1$ $(i=1,2,3,4)$.

With the same procedures as introduced above, we can realize the teleportation of this state. Firstly, Alice performs the Bell-state measurement on the particles x and 1, the corresponding states of particles y, 2, 3 and 4 are:

$$|\Psi_{mixed}\rangle_{y234}^{\boldsymbol{1}}=\sum_{i=1}^{4} P_i\ |\Psi_i\rangle_{y234}^{\boldsymbol{1}}\ \langle\Psi_i| \quad\Rightarrow\quad |\phi^+\rangle_{x1},\tag{23}$$

$$|\Psi_{mixed}\rangle_{y234}^{\boldsymbol{2}}=\sum_{i=1}^{4} P_i\ |\Psi_i\rangle_{y234}^{\boldsymbol{2}}\ \langle\Psi_i| \quad\Rightarrow\quad |\phi^-\rangle_{x1},\tag{24}$$

$$|\Psi_{mixed}\rangle_{y234}^{\boldsymbol{3}}=\sum_{i=1}^{4} P_i\ |\Psi_i\rangle_{y234}^{\boldsymbol{3}}\ \langle\Psi_i| \quad\Rightarrow\quad |\psi^+\rangle_{x1},\tag{25}$$

$$|\Psi_{mixed}\rangle_{y234}^{\boldsymbol{4}}=\sum_{i=1}^{4} P_i\ |\Psi_i\rangle_{y234}^{\boldsymbol{4}}\ \langle\Psi_i| \quad\Rightarrow\quad |\psi^-\rangle_{x1},\tag{26}$$

where $|\Psi_i\rangle_{y234}^{\boldsymbol{j}}$ $(j=1,2,3,4)$ has the same form as the above states of (6), (7), (8) and (9), and the only difference is to replace the coefficients x_1, x_2, x_3 and x_4 with x_{1i}, x_{2i}, x_{3i} and x_{4i}. Then Alice informs Bob of her measurement result. With cooperated unitary transformations ($I^2 \otimes I^3$, $\sigma_z^2 \otimes I^3$, $\sigma_x^2 \otimes \sigma_z^3$ or $i\sigma_y^2 \otimes \sigma_z^3$), the state of particles y, 2, 3 and 4 can be always transformed into $|\Psi_{mixed}\rangle_{y234}^{\boldsymbol{I}}$. Then Alice performs the second Bell-state measurement on particles y and 3, the state composed of particles 2 and 4 which are kept in Bob's hand would collapse onto one of the following states:

$$|\Upsilon_{mixed}\rangle_{24}^{\boldsymbol{A}} = \sum_{i=1}^{4} P_i \, |\Upsilon_i\rangle_{24}^{\boldsymbol{A}} \langle\Upsilon_i| \quad \Rightarrow \quad |\phi^+\rangle_{y3}, \tag{27}$$

$$|\Upsilon_{mixed}\rangle_{24}^{\boldsymbol{B}} = \sum_{i=1}^{4} P_i \, |\Upsilon_i\rangle_{24}^{\boldsymbol{B}} \langle\Upsilon_i| \quad \Rightarrow \quad |\phi^-\rangle_{y3}, \tag{28}$$

$$|\Upsilon_{mixed}\rangle_{24}^{\boldsymbol{C}} = \sum_{i=1}^{4} P_i \, |\Upsilon_i\rangle_{24}^{\boldsymbol{C}} \langle\Upsilon_i| \quad \Rightarrow \quad |\psi^+\rangle_{y3}, \tag{29}$$

$$|\Upsilon_{mixed}\rangle_{24}^{\boldsymbol{D}} = \sum_{i=1}^{4} P_i \, |\Upsilon_i\rangle_{24}^{\boldsymbol{D}} \langle\Upsilon_i| \quad \Rightarrow \quad |\psi^+\rangle_{y3}, \tag{30}$$

where $|\Upsilon_i\rangle_{24}^{\boldsymbol{K}}$ ($K = A, B, C, D$) possesses the same form as the states of (12), (13), (14) and (15). Consequently, to reconstruct the initial bipartite mixed state, Bob can perform the same transformation gate U^A, U^B, U^C or U^D on the state of particles 2 and 4 ($|\Upsilon_{mixed}\rangle^A$, $|\Upsilon_{mixed}\rangle^B$, $|\Upsilon_{mixed}\rangle^C$ or $|\Upsilon_{mixed}\rangle^D$) in accord with the result of Alice's measurement ($|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$ or $|\psi^-\rangle$), respectively. And the task of teleportation of bipartite mixed state is thus completed.

In fact, if we regard the teleported mixed state as the "compressed density martrix" [13], then the teleportation of the mixed state would reduce to the teleportation of an ensemble of pure states, which could also be equal to the sum of individual teleportations of pure states using the above method [14].

Now we would like to calculate the entanglement (or entanglement of formation) of the distributed cluster state and evaluate its ability for quantum teleportation. We follow the formulas in [11]. Consider a density matrix ρ ($\rho = |\varphi\rangle\langle\varphi|$) composed of a pair of quantum systems A and B, the entanglement is defined as the entropy of either of the two subsystems A and B:

$$\mathcal{E}(\varphi) = -\mathbf{Tr}(\rho_A \log_2 \rho_A) = -\mathbf{Tr}(\rho_B \log_2 \rho_B), \tag{31}$$

where ρ_A (ρ_B) is the partial trace of $|\varphi\rangle\langle\varphi|$ over subsystem B (A). The formula for entanglement makes use of what can be called the spin flip transformation, a function applicable to states of an arbitrary number of qubits. For a single qubit, the spin flip is defined by

$$|\varphi^s\rangle = \sigma_y|\varphi^*\rangle, \tag{32}$$

where $|\varphi^*\rangle$ is the complex conjugate of $|\varphi\rangle$ when it is expressed in a fixed basis such as $\{|0\rangle, |1\rangle\}$ and $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. And as to a spin flip on n qubit, we can apply the above transformation to each individual qubit. Then the entanglement of the state $|\varphi\rangle$ can be calculated as

$$\mathcal{E}(\varphi) = \mathcal{F}(|\langle\varphi|\varphi^s\rangle|), \tag{33}$$

where the function $\mathcal{F}$ is given by

$$\mathcal{F}(x) = -\frac{1+\sqrt{1-x^2}}{2}\log_2\left(\frac{1+\sqrt{1-x^2}}{2}\right) - \frac{1-\sqrt{1-x^2}}{2}\log_2\left(\frac{1-\sqrt{1-x^2}}{2}\right). \tag{34}$$

Similarly, the formula for the entanglement of formation of a mixed state ρ is

$$\mathcal{E}(\rho) = \mathcal{F}(C(\rho)), \tag{35}$$

where

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{36}$$

in which λ_i ($i = 1, 2, 3, 4$) are the eigenvalues, in decreasing order, of the Hermitian matrix

$$R \equiv \sqrt{\sqrt{\rho}(\sigma_y \otimes \sigma_y\ \rho\ \sigma_y \otimes \sigma_y)\sqrt{\rho}}. \tag{37}$$

Note that each ρ here is a non-negative real number.

When the four-particle cluster state (ρ_{1234}) is distributed to Alice and Bob for two particles each (1 and 3 to Alice, 2 and 4 to Bob), they would share the cluster entanglement. Because the communication parties apply Bell-state measurement twice in the process of teleportation, to evaluate the teleportation capability, we separate the cases and calculate the entanglement they use each time. At the first time, the entanglement they use can be viewed as between particles 1 and 2 (or 4) and the reduced density of operator in Alice's side is a mixture

$$\begin{aligned} \rho_{first} &= \mathbf{Tr}_{34}(\rho_{1234}) \\ &= \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|. \end{aligned} \tag{38}$$

So we obtain

$$\lambda_{first1} = \frac{1}{2}, \qquad \lambda_{first2} = 0, \qquad \lambda_{first3} = 0, \qquad \lambda_{first4} = 0. \tag{39}$$

Substituting the results into (35) gives us

$$\mathcal{E}(\rho_{first}) = -\frac{2+\sqrt{3}}{4}\log_2\left(\frac{2+\sqrt{3}}{4}\right) - \frac{2-\sqrt{3}}{4}\log_2\left(\frac{2-\sqrt{3}}{4}\right). \tag{40}$$

For the second time, while regarding as they take advantage of the entanglement between particle 3 and 4 (or 2), doing the calculation in full analogy, we obtain

$$\begin{aligned} \rho_{second} &= \mathbf{Tr}_{12}(\rho_{1234}) \\ &= \frac{1}{2}|01\rangle\langle 01| + \frac{1}{2}|10\rangle\langle 10|, \end{aligned} \tag{41}$$

and

$$\lambda_{second1} = \frac{1}{2}, \qquad \lambda_{second2} = 0, \qquad \lambda_{second3} = 0, \qquad \lambda_{second4} = 0, \tag{42}$$

resulting in

$$\mathcal{E}(\rho_{second}) = -\frac{2+\sqrt{3}}{4}\log_2\left(\frac{2+\sqrt{3}}{4}\right) - \frac{2-\sqrt{3}}{4}\log_2\left(\frac{2-\sqrt{3}}{4}\right). \tag{43}$$

Therefore, we can see it is the entanglement of cluster state $\mathcal{E}(\rho_{first})$ and $\mathcal{E}(\rho_{second})$ that provide the probability to realize the successful teleportation.

3 Conclusion

In this paper, we proposed an efficient and deterministic protocol for teleportating arbitrary two-particle pure and mixed state with cluster state. Compared with Deng et al.'s method using two three-particle GHZ state [6], our protocol is more economic since only four particles are required to be shared each time. It is also worth mentioning that the previous protocol using cluster state for teleportation [7] has to introduce the auxiliary qubit, and in our protocol this condition is not necessary. As a result, our protocol is more convenient and applicable.

We have shown the benefit of the cluster state channel for teleportation of arbitrary bipartite states with its inner particular feature of cluster entanglement. Future work should be focused on high dimensional or multiparticle state teleportation through this kind of quantum channel. With those good features of cluster state, more economic and useful protocols are expected to be upcoming.

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